12-Fold Interlaced Rosette – by Amina Quraish

Islamic Art has three major components: Arabic calligraphy, geometric art and biomorphic patterns. Geometric patterns are the most widely recognized visual tradition from the Islamic world. Islamic geometric art began in the 9th century as simple patterns with stars and diamond shapes. The art evolved to include patterns with six-fold to 13-fold patterns by the 13th century. By the 16th century Islamic geometric art was at its pinnacle and included 14 and 16-pointed stars in geometric compositions. A pattern can be classified into 'folds' by the number of lines of symmetry it has. With most geometric patterns the number of folds can be easily identified by counting the number of points in the center star. Islamic geometric patterns can be constructed using just a compass and ruler and do not require any complex mathematical calculations. These patterns are continuous and can grow infinitely.

How was this Islamic geometric pattern constructed? By looking at a geometric composition, it may not be obvious how it was constructed. The final outcome is visible, but not the steps taken to create it. In my 12-fold rosette painting, I started by first drawing a circle, the foundation of all geometric patterns. Then I divided the circle into 12 equal parts, which are the 12-folds of symmetry in the pattern. Within the circle, using the division by twelve, I drew two interlacing hexagons. Then I drew more intersecting lines until the final pattern emerged. After embellishing the pattern by painting gold interlacing lines, I decided to leave the construction lines to show the creative process that takes place before the final pattern is created. This pattern can be identified as a 12-fold pattern by looking at the center star, which has twelve points.

Nancy Blachman with Bill Gosper and Howard Cannon
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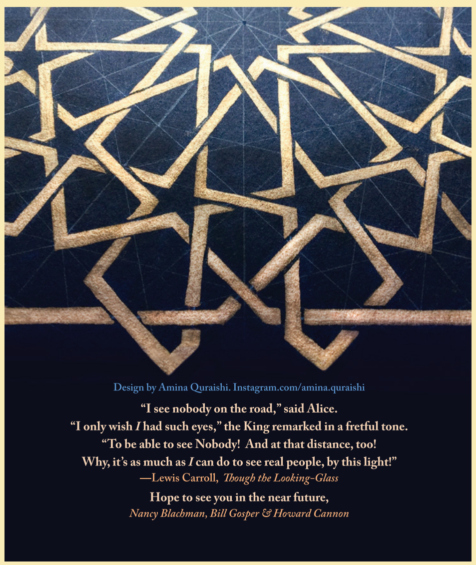

History of This Cloth

Microfiber cloths came to mind when I thought about creating a useful G4G13 exchange gift. A microfiber cloth can be easily folded, inserted into a pocket, purse, case, or book, and used to clean glasses or screens, something that many people have these days.

Bill Gosper suggested scores of mathematical designs for the cloth. Unfortunately, none of them caught my fancy. So I looked on the Internet and came across a poster for a workshop on Traditional Islamic Art. Amina Quraishi, the instructor and designer, granted me permission to reprint her artwork.

Bill Gosper continued sending me designs. The triangular fractal design caught my eye, so I decided to print it on one side of the cloth. The design was created by Julian Ziegler Hunts, one of Bill Gosper's protégés, with his Fourierizer. Howard Cannon created an interactive version of it. Learn about it at howwords.com/g4g.

Quotes – Next I turned my attention to finding or creating two thought-provoking quotes, one for each side of the cloth. Since Martin Gardner wrote *The Annotated Alice*, I asked Mark Burstein, the editor of its new edition and president emeritus of the *Lewis Carroll Society of North America*, for some of his favorite quotes. I settled on the following two:

> I see nobody on the road,' said Alice.
> "I only wish *I* had such eyes,' the King remarked in a fretful tone.
> 'To be able to see Nobody! And at that distance, too!
> Why, it's as much as *I* can do to see real people, by this light!"
> **—Lewis Carroll, *Alice's Adventures in Wonderland***
>
> "Would you tell me,
> please, which way I ought to go from here?"
> "That depends a good deal on
> where you want to get to," said the Cat.
> "I don't much care where—" said Alice.
> "Then it doesn't matter which way you go," said the Cat.
> "—so long as I get *somewhere*,"
> Alice added as an explanation.
> "Oh, you're sure to do that," said the Cat,
> "if you only walk long enough."
> **—Lewis Carroll, *Alice's Adventures in Wonderland***

Because we planned to give away the cloths to attendees of the 13th Gathering 4 Gardner (G4G13) and public events sponsored by G4G, we decided to replace the longer quote with one from the Introduction of Martin Gardner's book *Mathematical Carnival*.

Introduction

A TEACHER of mathematics, no matter how much he loves his subject and how strong his desire to communicate, is perpetually faced with one overwhelming difficulty: How can he keep his students awake?

The writer of a book on mathematics for laymen, no matter hard he tries to avoid technical jargon and to relate his subject-matter to reader interests, faces a similar problem: How can he keep his readers turning the pages?

The "new math" proved to be of no help. The idea was to minimize rote learning and stress "why" arithmetic procedures work. Unfortunately, students found the commutative, distributive, and associative laws, and the language of elementary set theory to be even duller than the multiplication table. Mediocre teachers who struggled with the new math became even more mediocre, and poor students learned almost nothing except a terminology that nobody used except the educators who had invented it. A few books were written to explain the new math to adults, but they were duller than books about the old math. Eventually, even the teachers got tired of reminding a child that he was writing a numeral instead of a number. Morris Kline's book. *Why Johnny Can't Add*, administered the *coup de grâce*. The best way, it has always seemed to me, to make mathematics interesting to students and laymen is to approach it in a spirit of play. On upper levels, especially when mathematics is applied to practical problems, it can and should be deadly serious. But on lower levels, no student is motivated to learn advanced group theory. for example, by telling him that he will find it beautiful and stimulating, or even useful, if he becomes a particle physicist. Surely the best way to wake up a student is to present him with an intriguing mathematical game, puzzle, magic trick, joke, paradox, model, limerick, or any of a score of other things that dull teachers tend to avoid because they seem frivolous.

No one is suggesting that a teacher should do nothing but throw entertainments at students. And a book for laymen that offers nothing but puzzles is equally ineffective in teaching significant math. Obviously there must be an interplay of seriousness and frivolity. The frivolity keeps the reader alert. The seriousness makes the play worthwhile. That is the kind of mix I have tried to give in my *Scientific American* columns since I started writing them in December, 1956. Six book collections of these columns have previously been published. This is the seventh. As in earlier volumes, the columns have been revised and enlarged to bring them up to date and to include valuable feedback from readers.

The topics covered are as varied as the shows, rides, and concessions of a traveling carnival. It is hoped that the reader who strolls down this colorful mathematical midway, whether he is "with it" as a professional mathematician or just a visiting "mark," will enjoy the noisy fun and games. If he does, he may be surprised, when he finally leaves the lot, by the amount of nontrivial mathematics he has absorbed without even trying.

MARTIN GARDNER
April, 1975

This quote is too long to include on the cloth, so we pared it down to the highlighted portions.

> "Surely the best way to wake up a student is to
> present him with an intriguing mathematical game,
> puzzle, magic trick, joke, paradox, model, limerick… .
> It is hoped that the reader who strolls down this colorful
> mathematical midway … will enjoy the noisy fun and games.
> If he does, he may be surprised … by the amount of nontrivial
> mathematics he has absorbed without even trying."
>
> **—Martin Gardner, Introduction to *Mathematical Carnival***

Designs – Thanks to Julian Ziegler Hunts, with assistance from Howard Cannon and Bill Gosper, for the triangular design. Julian constructed a spacefill dense with quadruple points, whose Fourier series Howard Cannon animated with his Pixel perfect Ptolemizer playpen.

The playpen was based on a "sandbox" with a general Fourier animator that Bill created. It takes a formula for the coefficients (amplitudes and phases as functions of frequency), speed, and coloration, and then creates a drawing. It's quite slow because it draws one pixel at a time, emulating the Symbolics mathematically correct draw-triangle ALU-add microcode necessary for smoothly drawing moving edges. Mathematically correct means that, if two triangles share an edge, there will never be missing or overwritten pixels, and if the triangle is "inside out," the "bump" will be negated. (The trick is to define consistently exactly which pixels to bump.) With this primitive, you can then animate the movement of an edge segment simply by drawing two triangles filling the quadrilateral defined by the old and new positions of its endpoints. To draw a seamless T joint, just include the "zero area" triangle formed by the constituent edges, which will usually add or subtract the "bump" constant to or from a few pixels.

Howard implemented a fast, full-res re-creation of the Symbolics triangle primitive, along with several lovely Ptolemaic sweeps, plus luxurious controls: e.g., for a nice picture of 2D Gibbs ringing, change the "Circle" box to Gibbs Tri, and click >. Besser illustrates frequency (over)modulation. The vector sum runs back and forth <Amplitude> radians along the circumference of the circle.

If you vary the number of its rotors on Howard's Ptolemaic Playpen page, you see the first few hundred Fourier approximations to a fairly standard triangle-filling function. This (#200 or 201) was particularly elegant. If you quadruple the number of rotors (harmonics) you see the next level of recursion. Julian creates absolutely magic *Mathematica* programs to evaluate the exact function and (incredibly) its multivalued inverse, as well as its Fourier series. True triangle-filling functions continuously map [0,1] **onto** a triangle, covering **all** its points, a dense subset of them at least three times! E.g., if you give Julian ½ + i^⅔, his program will quickly find the four preimages {3/8, 11/24, 9/16, 17/24}.